

Applications of Bessel functions in Physics and Engineering

One notes also that [Bessel's equation](#) arises in the derivation of separable solutions to [Laplace's equation](#), and also for the Helmholtz equation in either cylindrical or spherical coordinates. The [Bessel functions](#) are therefore very important in many physical problems involving [wave](#) propagation, wave diffraction phenomena-including [X-ray diffraction](#) by certain molecular crystals, and also [static](#) potentials. The solutions to most problems in cylindrical coordinate [systems](#) are found in terms of [Bessel functions of integer order](#) ($\alpha = n$), whereas in spherical coordinates, such solutions involve Bessel functions of half-integer orders

($\alpha = n + 1/2$). Several examples of Bessel function solutions are:

1. The diffraction pattern of a helical [molecule](#) wrapped around a cylinder computed from the [Fourier transform](#) of the helix in cylindrical coordinates;
2. Electromagnetic waves in a cylindrical waveguide
3. Diffusion problems on a lattice.
4. Vibration modes of a thin circular, tubular or annular membrane (such as a drum, Other membranophone, the vocal cords, etc.)
5. [heat conduction](#) in a cylindrical [object](#)

In engineering Bessel functions also have useful properties for signal processing and filtering noise as for example by using Bessel filters, or in FM synthesis and windowing signals.

Applications of Bessel functions in Physical Crystallography

The first example listed above was shown to be especially important in molecular biology for

the structures of helical secondary structures in certain proteins (e.g. α - *helix*) or in molecular genetics for finding the double-helix structure of Deoxyribonucleic Acid ([DNA](#)) molecular crystals with extremely important consequences for genetics, biology, mutagenesis, molecular evolution, contemporary life sciences and medicine. This finding is further detailed in the next subsection.

X-Ray Diffraction Patterns of Double-Helical Deoxyribonucleic Acid (DNA)

Crystals

Francis C. Crick ([Nobel laureate in Physiology and Medicine in 1962](#)) published in *Acta Crystallographica* (1952;1953a,b) concise papers on X-ray diffraction patterns of a helix and coiled coils, respectively [[5,6,7](#)] in which he showed that such patterns can be completely described by the Bessel functions defined above. Thus, the equatorial, or 0-layer, line contained

diffraction intensities whose values were computed with the J_0 [Bessel function](#) of the first kind with $n = 0$. In fact, the entire X-ray diffraction, multiple diamond-like pattern of such helices, including those of the double helical [DNA molecule](#), could be completely computed by means of Bessel functions of different order for each layer line; note however that there have also been occasional [contenders to this analysis](#). In fact, these involve *Fourier-Bessel series* based on Bessel functions.

There are, however, marked differences between the [A- and B- DNA X-ray diffraction patterns](#) as shown by this [web link](#) which makes a comparison between the images published by H.R. Wilson [[15](#)]. The Bessel function and Fourier-Bessel series analysis is however only applicable to the analysis of A-DNA patterns, whereas the X-Ray diffraction/scattering pattern of the B-DNA form is much less tractable although it is the predominant hydrated form in living cells.

The following is a web link to a [3D animation of a Watson-Crick DNA double-helix molecular model](#)

Note also that a pairing of double helices of a [DNA G-quadruplex](#) has also been recently discovered that might be associated with the initiation of certain cancers; the [square](#) of the Fourier transform of such [DNA G-quadruplex structures](#) would still result in diffraction patterns constructed from Bessel functions but the new quadruplex symmetry of the 'mutated' DNA [G-quadruplex](#) would naturally alter the overall diffraction pattern intensities.

Further details and implications for both genomic and biotechnology applications are presented in a related entry on [molecular models of DNA](#).

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The X-ray patterns of A- and B- DNA forms are compared in the following linked image ([courtesy of Dr. H.R. Wilson, F.R.S.](#))

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